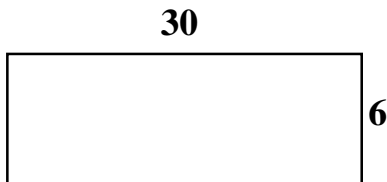# Math*Stars*

★★★ 1. How many 2's must be multipled to-gether for the product to be a number between 100 and 200?
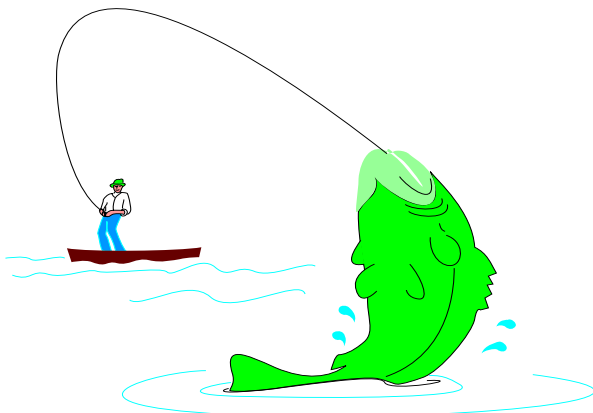
Answer: _____

★★ 2. How many 2 x 5 tiles are needed to cover this floor?

30

6

Answer: _____

★★ 3. At 9:00 a.m., I went to the Ol' Fishin' Hole to fish. There is a three fish per hour limit. If I need 20 fish for a cook-out tomorrow, at what time will I probably have my 20 fish?

Answer: _____

★★★ 4. Mary has three skirts, two blouses, and either black or white shoes that she likes to wear to school. How many days can she go without repeating the same combination of skirt, blouse, and shoes?

Answer: _____
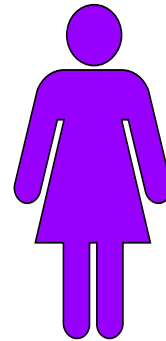
Strategy of the Month

Someone said, "A picture is worth a thousand words." Turning the words of a problem into a picture or a diagram can help you "see" the problem. By using the part of your brain that visualizes a situation or object, you may see relationships or information that helps you solve the problem. When someone tells you a story, try turning the words into a motion picture or a cartoon. When reading a descrip-tion, try "seeing it in your mind's eye." If you can do these things, this strategy may be for you! Try using a picture or make a diagram to solve this problem:

In the restaurant there are 12 square tables. Only one person can sit on each side. What is the greatest number of people that can be seated if the tables are pushed end to end into one large rectangle?

★★★ 5. How many cubes do you think it will take to make a cube that is twice as high as one cube?

Answer: _____
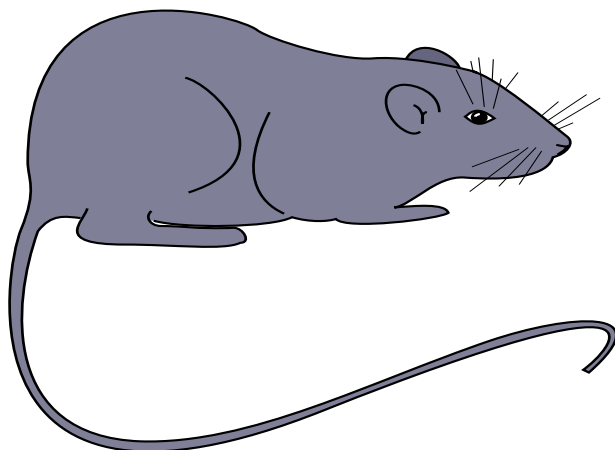
Three times as high?

Answer: _____

Four times as high?

Answer: _____

★★ 6. If a cat catches seven mice in four days, how many mice should it catch in 16 days?

Answer: _____

★★★★ 7. At the end of the soccer tournament, each team captain shakes hands with every other team captain. If there were eight teams in the tournament, how many handshakes were there?

Answer: _____

★ 8.

$$4\,3\,2$$
$$\underline{x\quad ?}$$
$$4\,7\,5\,2$$

? = _____

★★ 9. Julia spent 1/3 of her birthday money. Then she lost 1/2 of the rest. She now has $10 left. How much did she get for her birthday?

Answer: _____

Math*Stars*

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As with all good problems, the solutions and strategies suggested are merely a sample of what you and your students may discover. Enjoy!!

Discussion of the problems...

 1. **(7)** Since 2x2x2x2x2x2x2=128; six would not be enough and eight would be too many.

2. **(18)** Students need to draw a figure or use rectangular tiles to model this solution. They should note that on the width or "six" side, three tiles of measure "2" will fit. On the lenght or "30" side six tiles of measure "5" will fit. Thus they will use 18 two by five tiles to cover the floor.
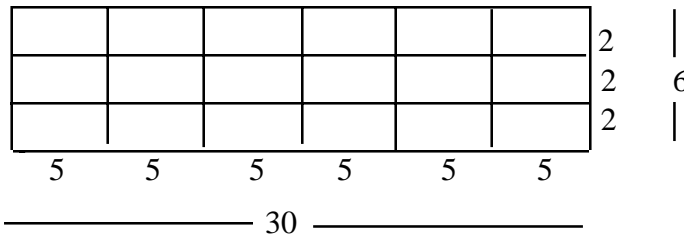
 3. **(4:00 PM)** Creating a chart is a good way to organize the information and determine a solution.

i.e.	Time	Catch	Total
	9:00-10:00	3 fish	3 fish
	10:00-11:00	3 fish	6 fish
	11:00-12:00	3 fish	9 fish
	•		
	•		
	•		

During the 3:00-4:00 hour, the 20th fish will be caught.

4. **(12)** A tree diagram is one strategy which students may employ to solve this problem. Colored markers, cubes or other manipulatives can also help students model the situation.

```
                              Black shoes
                 Blouse #1
   Skirt #1                   White Shoes

                 Blouse #2
                              Black Shoes



                              White Shoes
                 Blouse#1
   Skirt #2                   Black Shoes
                              White Shoes
                 Blouse #2
                              Black Shoes



                              White Shoes
                 Blouse #1
   Skirt #3                   Black Shoes
                              White Shoes
                 Blouse #2
                              Black Shoes
```
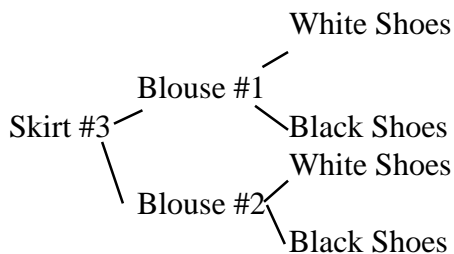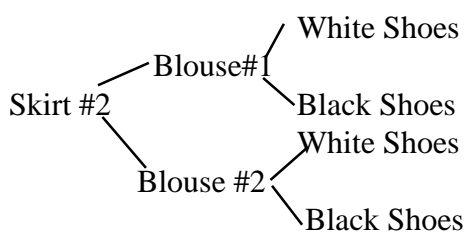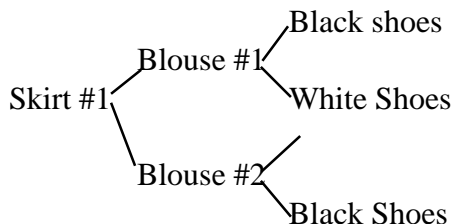
5. **(8, 27, 64)** Students can use unifix cubes to model the problems, especially if abstract visualization is difficult. Twice as high will be 2 x 4, three times as high 3 x 9, and four times as high would be 4 x 16.

6. **(28)** If he catches seven mice in four days, students can expansd to twice as many in eight days or, 14 mice. Doubling again, he can catch 28 in sixteen days. Another approach would be to create a table that increases by four days at each step (i.e. 7 mice in 4 days, 14 mice in 8 days, 21 mice etc.).

7. **(28)** Encourage students to make a chart to determine the combinations of handshakes.   They should note that when coach X shakes hands with coach Y it counts as a handshake for coach Y. Another strategy is modeling using the vertices of an octagon.

8. **(11)** Examining the problem students may initially determine that the missing multiplier is greater than ten (432 x 10 = 4320) How much greater than 10? 4752 - 4320 = 432 So one greater than ten or eleven. Another strategy could involve straight division.

9. **($30)** Working backwards is an excellent strategy to use in this situation. If losing half left her with $10, then she had $20 before that happened. If spending one-third left her with $20, then that must be two-thirds. She had three-thirds or all to start with and that must be $30.

Math*Stars*

★ 1. RIDDLE ME THIS:
   I'm thinking of a number.
   It is odd.
   It's between 1 and 100.
   It's higher than 20.
   It is smaller than the answer to 6 x 6.
   It is a multiple of 5.
   The sum of its digits is 7.

   Answer: _____

★★★ 2. Hank had an average of exactly 84% after taking two tests. On the third test, he scored 96%. Find his average for all three tests.

   Answer: _____

★★★ 3. What day of the week was yesterday, if five days before the day after tomorrow was Wednesday?

   Answer: _____

S	M	T	W	Th	F	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

★★ 5. Complete the following number pattern:

   14 28 18 36 26 52 42 84 ___ ___ ___

   Answer: _____

★★ 6. You know that the perimeter of a certain rectangle measures 22 in. If its length and width each measure a whole number of inches, how many different areas (in square inches) are possible for this rectangle?

   Answer: _____

Strategy of the Month

*Your brain is an organizer. It organizes information as it stores that information. When a problem involves many pieces of information, your brain will have an easier time sorting through it if you make an organized list. A list helps you be sure you have thought of all of the possibilities without repeating any of them. Like drawing a picture or making a diagram, making an organized list helps your brain "see" the problem clearly and find a solution. Try **making an organized list** to solve this problem:*

Tickets for the concert cost $12 for adults or teenagers and $6 for children. If the group has $60, how many adults or teenagers and how many children could go?

★★★★ 6. Each salad contains red beans, lima beans, and black-eyed peas. Use the information below and determine how many of each of the three types of beans are needed.

 a. This salad contains at least 12 beans. It has one more lima bean than red beans. It has one more red bean than black-eyed peas.

 b. This salad contains the same number of red and lima beans. It has three more black-eyed peas than red beans. It has a total of 18 beans.

 c. Lima beans make up 1/2 of this salad. The salad has exactly two red beans. The number of lima beans is double the red beans.

★★★ 7. Suppose two hens lay three eggs in four days. At the same rate, how long will it take eight hens to lay a dozen eggs?

 Answer: _____
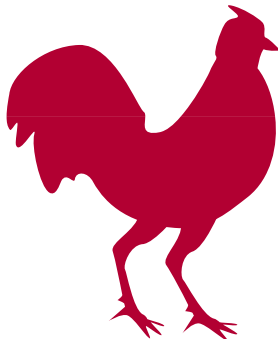
★★★ 8. The sale is 1/4 off the marked price. How much will you pay for this soccer ball, including tax of $.05 per dollar?
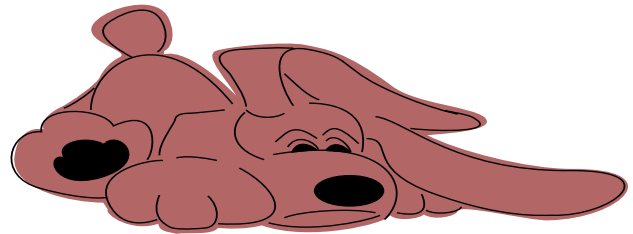
 Answer: _____

$ 20.00
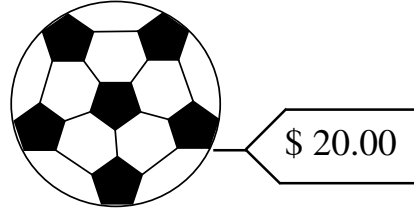
★★★ 9. Sam's father gave him 49 dog biscuits to give to their three dogs – Alice, Joe, and Mickey. Sam must divide the biscuits according to the size of the dogs. Joe needs to get twice as much as Alice and Mickey needs to get twice as much as Joe. How many biscuits should Sam give each dog?

 Answers:

 Alice: ____ Joe: _____ Mickey: _____

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As with all good problems, the solutions and strategies suggested are merely a sample of what you and your students may discover. Enjoy!!

Discussion of the problems...

1. **(25)** Students should satisfy one condition of the riddle at a time and eliminate numbers as the information is given. They can then be challenged to write riddles of their own to share with classmates.

 2. **(88)** Be sure all students have a good understanding of average. Unifix cubes can be used to demonstrate the "equal"ing or "even"ing of values when an average is computed. The formula will also be of help. Since 84 is his average after two tests the sum of those two scores is 2 x 84 or 168. When the third score is added the new average becomes 264 ÷ 3 or 88.

 3. **(Friday)** Encourage students to use the calendar to try out their conjectures. A similar problem : What will tomorrow be if yesterday was the 2nd day of the week? [Wednesday]

 4. **(74, 148, 138)** Start with an easier problem such as 2,   4,   3,   5,   4,   6,   5, . . . This is especially helpful if students have not seen series with two

or change factors   +2   -1 +2   -1   +2   -1
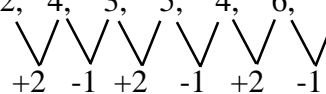
5. **(5)** Since the perimeter of the rectangle measures 22 inches, the sum of a single length and a single width must be half 22 or 11 inches. Students can make a table of possible lengths, widths and corresponding areas.

| length		10	9	8	7	6
width		1	2	3	4	5
area		10	18	24	28	30

6. **(a.) at least 3 black-eyed peas, 4 red beans and 5 lima beans; b.) 8 black-eyed peas, 5 red beans and 5 lima beans; c.) 2 black-eyed peas, 2 red beans and 4 lima beans)** Trial and error, guess and check are good strategies for this problem.. Tri-colored manipulatives will help students model their conjectures. This is also a good introduction to ratio and proportion.

7. **(4 days)** If 2 hens lay 3 eggs in 4 days, then 8 hens (four times as many) will lay 12 eggs in 4 days. Students can use drawings, charts or manipulatives to act out or model the situation.

8. **($15.75)** Students need to see the relationship between the regular price of $20 and the discount of 1/4 off. One-fourth of $20 is $5. When the regular price is reduced by $5 the new price become $15. Now for the tax. Tax is paid at the rate of 5 cents for each of 15 dollars for a total of 75 cents. Adding, the customer pays $15.75. If the tax is computed first, then it is $1, added to $20 gives us $21. Now take 1/4 off and the cost is still $15.75. This is a nice illustration of the commutative law for multiplication.

9. **(Alice - 7, Joe - 14, Mickey - 28)** Students need to begin with a careful reading of the problem and attention to details. A diagram would be helpful. Seeing that Alice gets the least, and that other portions are based on hers, will help students establish a method of attack. When guess and check are tried the total, 49, becomes the goal. Teachers may wish to lead students to this type of problem with a simpler example: Mary jumped 30 times before she missed a step in the jump rope contest. Sally jumped twice that number and Sam jumped half of what Mary jumped. How many times did Sally and Sam jump before they missed a step? [Sally - 60, Sam - 15]

Math*Stars*

★ 1. The length of one side of a regular hexagon is 20 cm. What is the perimeter of the hexagon?

Answer: _____

★★★★ 2. Given the number line below, express in fractional form the relationship between:
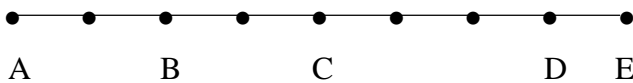
$\overline{AB}$ is what fractional part of $\overline{AD}$? _____

$\overline{AC}$ is what fractional part of $\overline{AD}$? _____

The length of $\overline{AB}$ is what percent of the length of $\overline{AE}$? _____

The length of $\overline{AB}$ is what percent of the length of $\overline{AC}$? _____

★★★ 3. A bag of marbles can be divided in equal shares among 2, 3, 4, 5, or 6 friends. What is the least number of marbles that the bag could contain?

Answer: _____

★★★ 4. Look at the picture below. Can the mouse reach its hole before the cat can catch it?

Answer: _____

Mouse runs 10 meters per second

Cat runs 20 meters per second
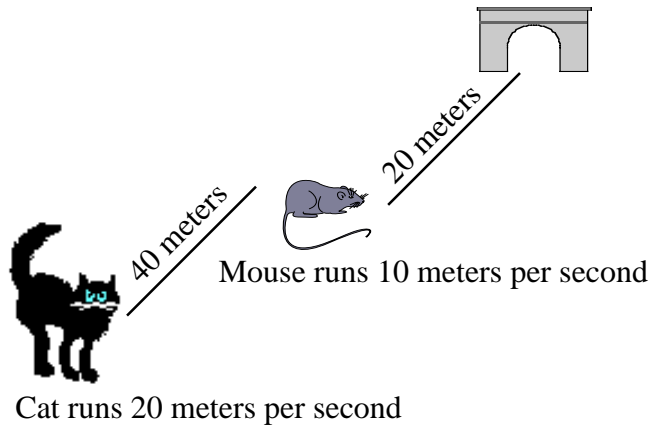
Strategy of the Month

Being a problem solver is something like being a detective! A detective has to solve crimes by guessing what happened and checking the guess to see if it fits the situation. For some problems, your best strategy may be to make a guess and then check to see if your answer fits the problem. If not, decide if your guess was too high or too low and then make a second "guesstimate." A good detective keeps records (usually some kind of chart) to help see any patterns and to narrow down the possibilities. You should do this too. The results of incorrect guesses can give you valuable clues to the correct solution. **Guess and then check** *the solution to this problem:*

Use exactly 50 coins to make one dollar. You must have at least one penny, one nickel, one dime, and one quarter.

★★★ 5. Michael was supposed to multiply a number by 5. By mistake, he divided the number by 5 instead. His answer was 5. What should have been the correct answer?

Answer: _____

★★ 6. The fifth grade is going on a field trip to the zoo. The zoo requires that for every 15 students, there must be one chaperon. If there are 194 students going on the trip, how many chaperones will be needed?
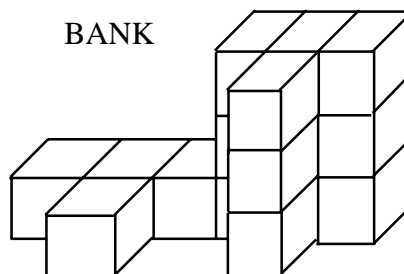
Answer: _____

★★★ 7. Find the volume for each building.
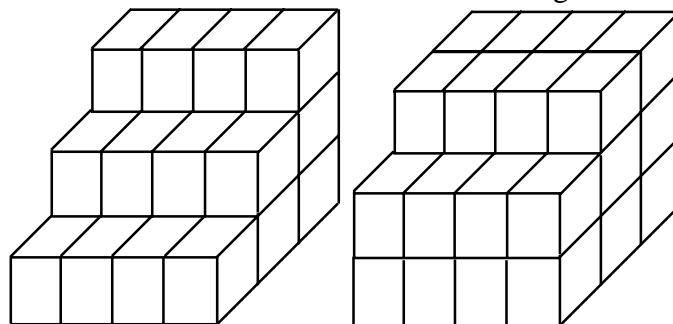
BANK   OFFICE

HOSPITAL

Bank _____

Hospital _____

Office _____

★★ 8. What is the greatest six-digit number in which the thousands place is twice the digit in the tens place? What is the least number?

Answer:

Greatest number _____

Least number _____

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Discussion of the problems...

1. **[120cm]** The diagram of the hexagon is not shown because students need to recall that a hexagon has six sides. Regular indicates all sides are congruent. Further application could involve finding the perimeter of a regular triangle, quadrilateral, pentagon, and octagon.

2. **(2/7, 4/7, 25%, 50%)** Students need to observe that segment AD is divided into seven equal parts. If segment AE represents a whole, and it is divided into eight equal parts, then AB is one-fourth or 25%. Considering AC as a whole, AB is half or 50%.

3.**(60)** The solution is the least common multiple (LCM) of 2, 3, 4, 5, and 6. Listing multiples of each and comparing is an excellent strategy.

 2--->2, 4, 6, 8, . . . 56, 58, **60**, 62, . . .
 3--->3, 6, 9, 12, . . .54, 57, **60**, 63, . . .
 4--->4, 8, 12, 16, 20, . . .52, 56, **60**, 64, . . .
 5--->5, 10, 15, 20, . . .50, 55, **60**, 65, . . .
 6--->6, 12, 18, 24, . . 48, 54, **60**, 66, . . . So the LCM is 60.

4. **(yes)** Even though the cat can run faster than the mouse, in one second the mouse will be half-way to his holeand the cat will be half-way to the mouse's former position. In two seconds the mouse will be at his hole while the cat will be where the mouse was at the start or 20 meters from the hole. This is a good problem for students to act out.

5. **(125)** The first question the student needs to ask is " if N ÷ 5 = 5, what is N?" When they have arrived at 25 for the original multiplicand, then 25 x 5 will give the correct answer.

6. **(13)** This problem is a good application of mathematics and the importance of common sense or logic in its use. Division (194 ÷ 15) will give (12), the number of groups of 15 for which a chaperone is needed, however the remainder or left-over students, (14) will also require a chaperone, therefore 13 caperones are needed.

7. **(24, 16, 32)** Students may wish to use cubes to build these models and account for the hidden cubes in each structure.

8. **(998,949; 100,000)** Teachers may wish to point out that there are no restrictions on using a digit more than once.

Math*Stars*

★ ★ ★ 1. Use the numbers 4 through 12 to fill in the circles. The numbers on each straight line must add up to 21.
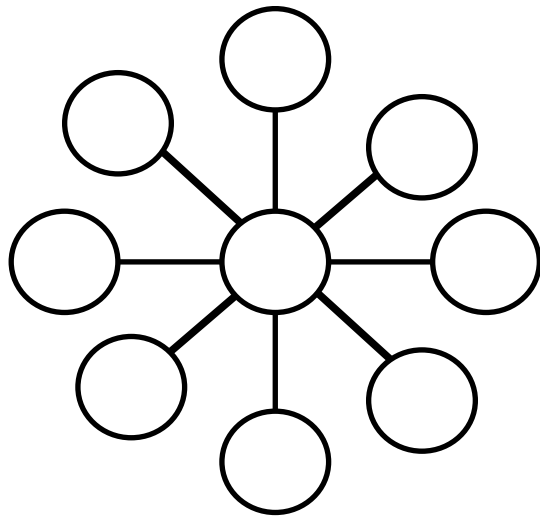
★ ★ ★ ★ 3. A rectangle lot 30m by 20m is surrounded on all four sides by a concrete walk 3m wide. If you need to concrete only the sidewalk, how much concrete will you need? (surface area)

Answer: _____
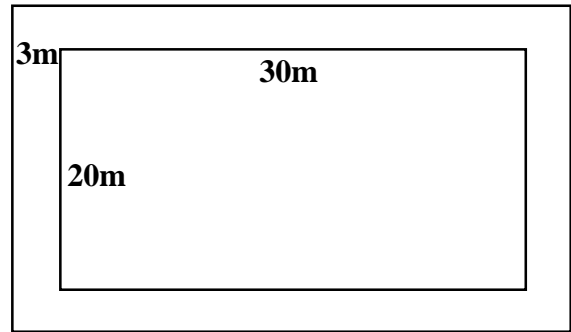
★ ★ 2. Your mother and father decide to change your allowance. You are given the choice:

a. They will pay you $10 a week
                    - or-
b. They will pay you one cent the first week, two cents the second week, four cents the next week, and so on, doubling your allowance each week for a year.

Which will give you the most money?

_____

Why?

Strategy of the Month

*Noticing patterns helps people solve problems at home, at work, and especially in math class! Math has been called "the study of patterns," so it makes sense to look for a pattern when you are trying to solve a problem. Recognizing patterns helps you to see how things are organized and to make predictions. If you think you see a pattern, try several examples to see if using the pattern will fit the problem situation. Looking for patterns is helpful to use along with other strategies such as make a list or guess and check. How can **finding a pattern** help you solve this problem?*

A palindromic number is onewhich reads the same backwards as forwards. How many 3-digit palindromic numbers are there?

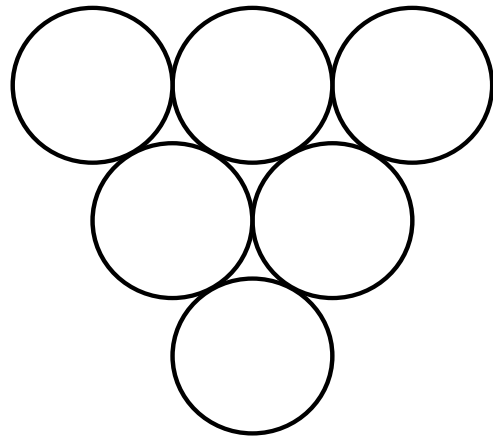★ 4. Label the correct measurements on the marked angles in the square.
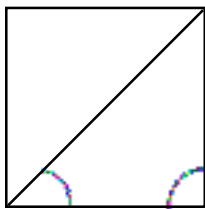
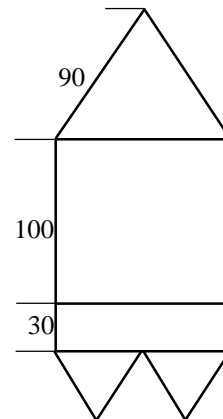★★ 5. What are the two least likely sums to be rolled on two regular dice? Why?

Answer: _____
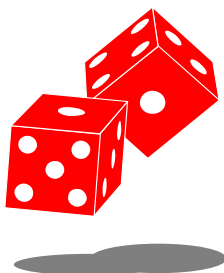
★★ 6. Move only two discs and turn the triangle upside down. (Draw arrows to show how to move them.)

★★★ 7. The figure below is constructed of equilateral triangles and rectangles. Label the ten unmarked segments with their correct lengths.

90

100

30

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Discussion of the problems...

 1. (**Answers will vary.**) One possible solution:
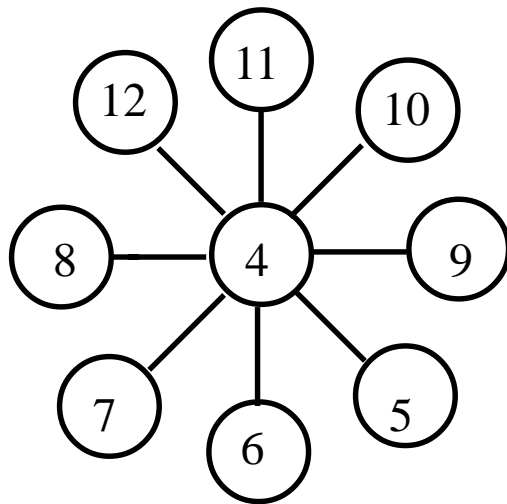
 2.(**b, doubling will surpass the fixed amount after the 11th week and the total after the 14th**)
Students should be encouraged to make a chart comparing the allowances for a number of weeks.

Week #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	$10	$10	$10	$10	$10	$10	$10	$10	$10	$10	$10	$10	$10	$10	$10
Sum A	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150
B	.01	.02	.04	.08	.16	.32	.64	1.28	2.56	5.12	10.24	20.48	40.96	81.92	163.84
Sum B	.01	.03	.07	.15	.31	.63	1.27	2.55	5.11	10.23	20.47	40.95	81.91	163.83	327.67

.

3. **(336 square meters)** There are several approaches to this problem. Students may compute the area of the entire lot and of the unpaved portion. Subtracting will give them the paved area. Another strategy would be to divide the paved area into four rectangles and add their areas.
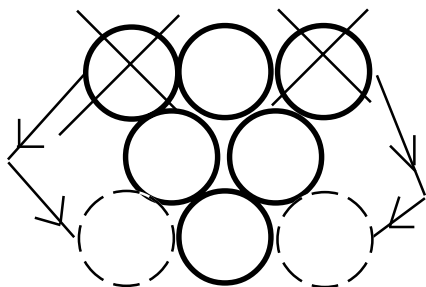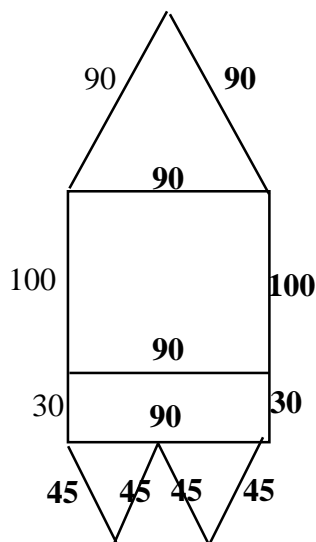
4.
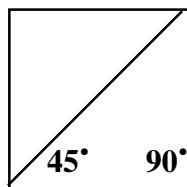
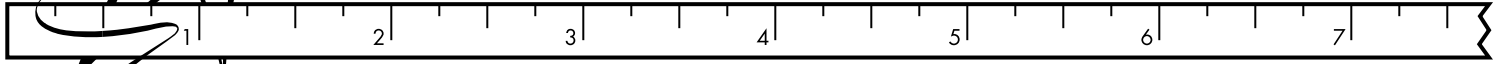Students may use protractors or logic to solve this problem.

45°   90°

5. **(2 and 12 because there is only one way to get these sums)** Students can best illustrate this situation with a chart of possible outcomes.

6.

7.

Math*Stars*

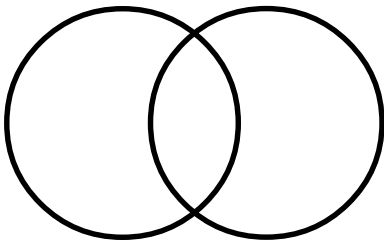*a problem solving newsletter*   ***Vol. 5 No. 5***

★★   1. If $x = 4$ and $y = 2$, then:

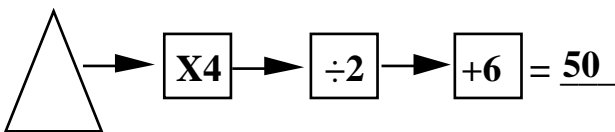$3x + y =$ _____ and

$4y - 2x =$ _____

★★   2. Graph the factors of 45 and 54 in the Venn diagram below.
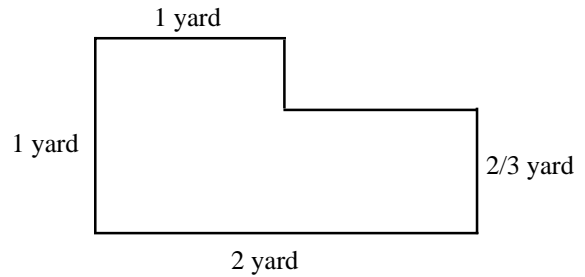
★★   3. If there are two computers for every 40 students at Elm Elementary, how many computers do they have for the 440 students attending school?

Answer: _____

★★   4. Write a number in the triangle that will make the answer 50.

★★★ 5. Find the area of the flower bed below in square feet.

Answer: _____

Strategy of the Month

*Sometimes mathematical ideas are hard to think about without something to look at or to move around. Drawing a picture or using objects or models helps your brain "see" the details, organize the information, and carry out the action in the problem. Beans, pennies, toothpicks, pebbles, or cubes are good manipulatives to help you model a problem. You can use objects as you guess and check or look for patterns. Try **using objects** to help you solve this problem:*

What happens to the volume of a rectangular prism if the width is tripled?
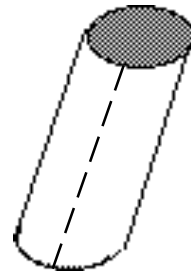
★★★ 7. On one night, 30 fifth graders gathered to study mathematics and science. Of these students, 11 studied math, 15 studied science, and 3 studied math and science. How many students of the group studied neither math nor science?

Answer: _____

★★★ 8. If you cut a cylinder along the dotted line and flatten it, the inside forms what shape?

Answer: _____

If the area of the flattened figure is 20 square inches and the distance around the top of the cylinder is 4", how tall is the tube?

Answer: _____

★ 9. There are 20 chickens, 4 horses, and 8 cows on the McDonald farm. How many legs are there?

Answer: _____

★★★★6. All of these are snobhops:

None of these are snobhops:

What is my rule: _____

Draw another snobhop:

Math*Stars*

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Discussion of the problems...

1. **(14, 0)** If students have not explored using unknowns and variables, concrete objects will be of help.   Put four objects in a bag and two in another. Show that 3x means three bags (with four in each) or 12 objects etc.

2. The diagram should be similar to the following:
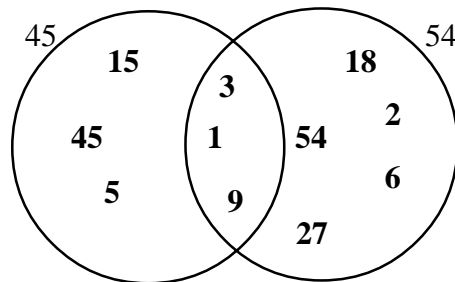
Be sure students understand the rationale for the common factors placement in the intersection of the Venn diagram.

3. **(22 computers)** If students have not yet mastered long division, there are other ways to solve this problem using number sense. Using diagrams or manipulatives the number of groups of 40 can be determined, followed by counting two computers per group. Another approach is to reason that two computers for 40 students means one computer for twenty. Then the number of groups of twenty can be calculated.

4. **(22)** Students can work backward to show that (50 - 6) x 2 ÷ 4 = 22. Some students may use guess and check or rely on their numbersense to solve the problem.

5. **(15 square feet)** The flower bed can be divided into two rectangles with either a horizontal or vertical line. Suggest that students change the measurements to feet before beginning.

6. **(Snobhops are sets of numbers which can be the lengths of the sides of triangles, i.e., the sum of <u>any</u> two is greater than the third.)** Students may wish to verify that snobhops always form triangles and that the non-snobhops cannot form triangles. Answers will vary for the student-generated snobhop but they should be ready to prove it fits the rule.
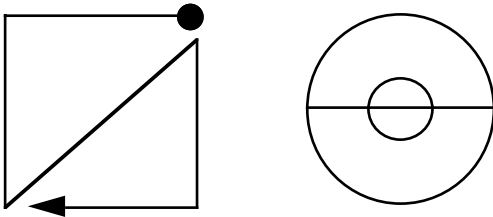
7. **(7)** Remind students that a Venn diagram is a very useful tool for sorting information in problems of this type. After they have completed the study groups for math and science, students should note that they have accounted for only 23 of the 30 students. This leaves seven who studied neither math nor science.
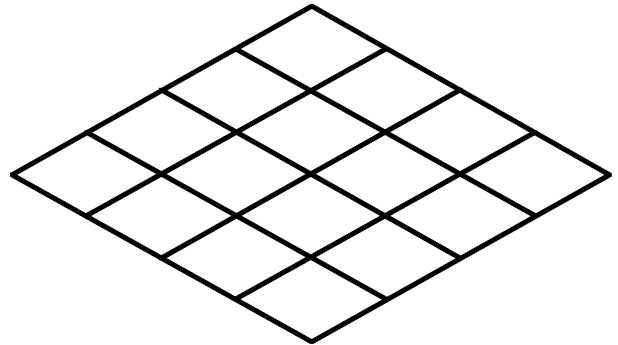
Study Group
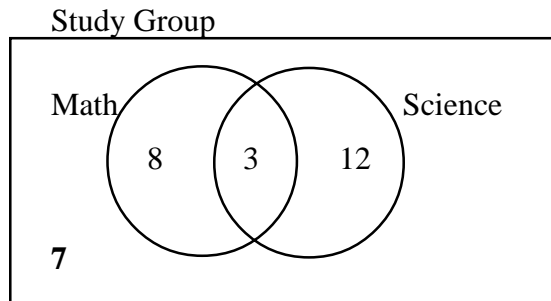
 8. **(rectangle; 5 inches)** If students have difficulty visualizing the figure, teachers may wish to use an empty paper towel roll to demonstrate that indeed a rectangle is the result. Knowing the area and the distance around the top (4 inches) students should be able to determine the length as,
 Area = length x width or 20 = 4 x ☐ .

9. **(88)** Since chickens have two legs and the other animals four legs each, this is a good opportunity for students to write expressions that illustrate the order of operations and the distributive and associative properties. 2 x 20 + 4 x 4 + 4 x 8 = 2 x 20 + (4 x 4 + 4 x 8) = 2 x 20 + (4 x [4 + 8]) = 2 x 20 + 4 x 12 = 40 + 48 = 88.

Math*Stars*

★   1. The square below can be traced with one continuous line without lifting a pencil or retracing a line. Find the correct path for the circle.

★★★★2. Juan received the following grades for the first grading period:

6	8, 8
7	1, 7, 7
8	3, 5, 7, 8
9	5, 9, 9, 9

Should Juan request that the teacher use the mean or median to determine his grade, if he has a choice? Why?

Answer: _____

★★   3. Using Roman numerals made from toothpicks, move one toothpick to make a true equation.

★★   4. Create a pattern using green, black, red, and yellow so that each color appears only once in every line of four diamonds.
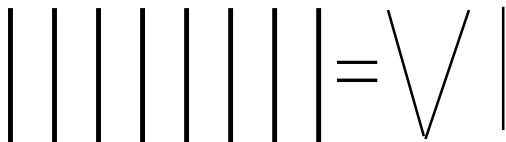

Strategy of the Month

*When a problem involves data with more than one characteristic, **making a table, chart, or graph** is a very good way to organize the information. It helps your brain to identify patterns and to discover any missing data. Tables help you record data without repeating yourself. Making a table or chart is especially useful for certain problems about probability and for some logic problems. Sometimes tables and charts are included in your information and you need to read through them carefully to understand the data you need to solve your problem. Creating a graph is also a good way to organize and visualize information. Make a table to solve this problem:*

Farmer Oakes had 15 animals in her farmyard. Some were chickens and some were cows. There were 52 legs in all. How many cows were in her farmyard?

★★   5. Arrange the digits 4, 8, 7, 2, and 9 such that the answer will be the largest possible product.
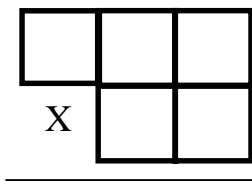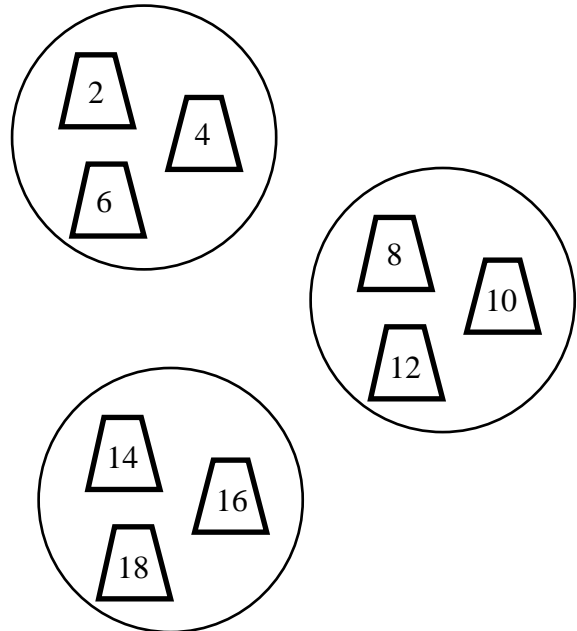

Answer: _____

★★   6. RIDDLE ME THIS:

    1 am > 5 x 10.
    I am < 100.
    I am an even number.
    I am not 70 or less.
    I am a multiple of 4.
    I am a multiple of 3.
    I am < 80.

Answer: _____

★★   7. Even up the weights in these circles by moving one weight to another circle. The sum of the weights in each circle should be equal.

★★★ 8. Five flags are evenly placed around a track. It took a runner 30 seconds to get from the first flag to the third flag. If the runner continues at the same speed, how long will it take her to get completely around the track?
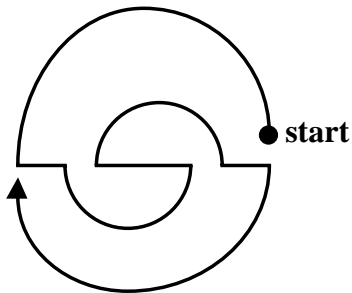
Answer: _____

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Discussion of the problems...

1.

2. **(He should choose the median because it is 85 while the mean is 84.)** Students will need to be able to read a stem and leaf plot, as well as know how to determine the mean and the median to solve this problem.

3. **(III + III = VI)** Some students may have forgotten about Roman numerals and the patterns they form. The fourth toothpick is laid across the fifth to form a plus sign!

4. One solution:
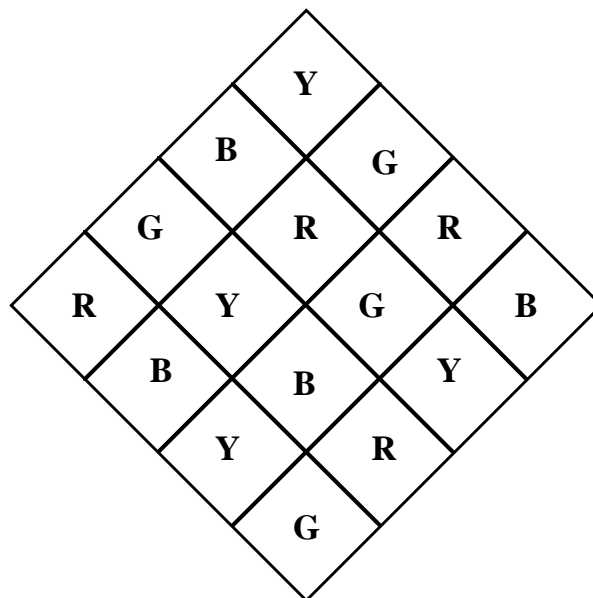
5. **(942 x 87 = 81,954)** Students should observe that the largest numerals are most powerful in the higher place value positions. Some students may use trial and error, but after several different examples, some should see the pattern and confidently place their numerals and explain their strategy. A good claculator problem and one that shows an application of place value.

6. **(72)** Students should be familiar with this type of problem and have developed a strategy for elimination. Checking an answer by "walking" through the clues is a good way to encourage verification. Students can make up problems of this type and try them out on their classmates.

7. **(move 18 to the circle with 2, 4, and 6)** Each circle will then total 30.

8. **(75 seconds or 1 one minute 15 seconds)** Encourage students to draw a diagram of the track and the placement of the flags. To run from the first to the third flag, the runner covers two intervals in 30 seconds. Each interval, therefore, takes her 15 seconds. Since there are five intervals, 5 x 15 =75. Seventy-five seconds can also be written as one minute 15 seconds.

Math*Stars*

a problem solving newsletter

★★   1. Square ABCD has one side of length 4 cm. Triangle BEC is an equilateral triangle. What is the perimeter of the figure ABECD?
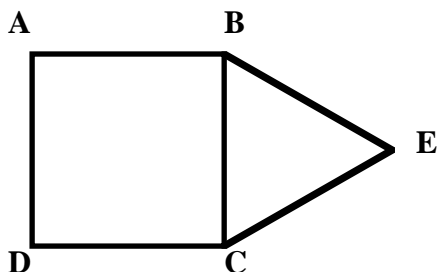
Answer: _____

★ ★ ★ ★ 4. Determine the area of this polygon if the segments have the indicated lengths.
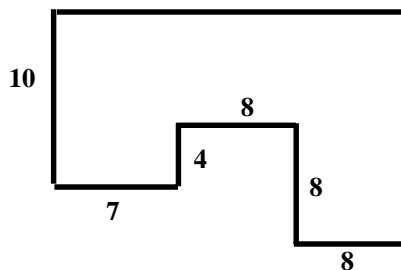
★   2. George, Susan, Henry, and Sarah are 7, 8, 9, and 10 years old. Henry is older than Sarah and younger than George. Susan is younger than Henry and older than Sarah. What is each person's age?

George's age _____

Susan's age   _____

Henry's age   _____

Sarah's age   _____

★ ★ ★ 3.

1000 x 100 x 10 x 1 x 0.1 x 0.01 x 0.001 =

Answer: _____

Strategy of the Month

Some problems are difficult to "see" even if you draw a picture. For these problems, it can be helpful to actually act out the problem. When you role-play with friends or people at home, you may discover the solution as you act out the problem. Or you may recognize another strategy that will help you find the answer. Sometimes "acting out" a problem can be done with manipulative materials. To find the solution to the problem below, become the director and choose your cast to act this out:

The students were in line at the movie theater to buy tickets. There was a student in front of two students, a student between two students, and a student behind three students. What is the least number of students that could have been in line?
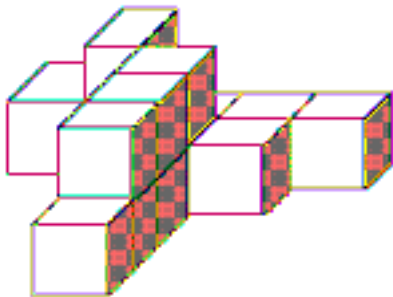
★★ 5. Farmer Brown had a garden plot that was two yards by three yards. He tripled its length and width the next year. What was the area of the new garden? Did the area triple?
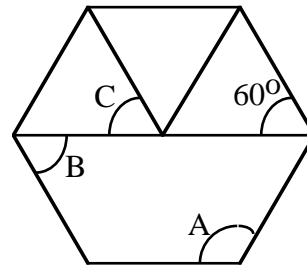
★★★ 6. How many seconds are in one week?

Answer: _____

★★★ 7. What is the volume of the figure below? If it were painted on the outside with purple, how many blocks would have exactly four faces painted purple?

★★★ 8. In a regular hexagon, what is the measurement of angle A, B, and C.

A _____ degrees

B _____ degrees

C _____ degrees

★★ 9. In these addends, each letter represents a distinct digit. Find the digits.

```
    C E N T        T = ____
    C E N T        E = ____
  + S C E N T      N = ____
  ───────────      C = ____
    3 5 1 2 8      S = ____
```

Math*Stars*

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Discussion of the problems...

1. (**20 cm**) Since the square and the equilateral triangle share a side, all outside dimensions are equal, 4 centimeters.

2. (**George is 10, Henry is 9, Susan is eight and Sarah is 7.**) One strategy that will help in solving this problem is trial and error or guess and check. Students can propose an age for one child and carry out the conditions to see if they are logical. This type of reasoning gives students a chance to pose possible solutions and refine their guesses until they have satisfied all the conditions in the problem. Manipulatives or drawings will help them keep track of their conjectures.

3. (**1**) Some students may first approach this problem by multiplying left to right. A second look at the factors might inspire (or they could be led to) an approach of organization. The commutative property of multiplication would allow a pairing of certain factors. Ten times one-tenth is? One hundred times one one-hundredth is? and so on. A closer look at reciprocals could engage students in creating their own mental math strings, e.g., 1/2 x 5 x 1/3 x 3/4 x 8 x 2 x 1/5 x 3 x 4/3 = ?

4. (**230 square units**) Students will need to view the figure as a series of rectangles and determine the lengths of unlabeled sides using logic. Reproducing the diagram on graph paper will also help students determine missing dimensions. This is a good problem to have students share and explain their methods to the entire class. Often, seeing a different strategy will inspire students to adopt it themselves. One 'elegant' solution moves the 8 by 4 rectangle at the bottom right to the gap in the middle. This gives a 10 by 23 rectangle which yields the area with mental calculation!

5. (**54 square yards, no**) The dimensions of the expanded garden are 6 by 9. Some students may wish to model or draw the plot to determine the new dimensions and compare the areas.

6. (**604,800 seconds**) This is a wonderful application of calculator use. The numbers are a bit cumbersome and can interfere with student's train of thought in plotting the calculations necessary. One week = 7 days = 7 x 24 hours = 7 x 24 x 60 minutes = 7 x 24 x 60 x 60 seconds.

7. **(volume 14 cubic units, 3 blocks)** It may help students if they can model the problem with blocks to see the figure in question.

8. **(<A: 120˚, <B: 60˚, <C: 60˚)** Pattern blocks will help students determine that the interior triangles are equilateral. From there they can reason that <A is two 60˚ angles and <B and <C are each 60˚.

9. **(T = 6, E = 3, N = 7, C = 8, S = 1)** Guess and check as well as logical reasoning about number values will help students solve this problem.   A starting help might be to consider which number added three times has a sum with an 8 in ones place, i.e., T+T+T= ⬚8.

★★★★1. In Mr. Higdon's class three out of five students prefer chocolate ice cream. Two out of five students are evenly divided in their preference for vanilla and strawberry. If one carton of ice cream will feed ten students, how many cartons of vanilla, strawberry, and chocolate ice cream will Mr. Higdon need in order to give his 25 students their choice of flavors for the class party? How much ice cream of each flavor will be left over?

	# ice cream cartons needed	left over
chocolate		
vanilla		
strawberry		

★★   2. If you could fold a paper in half six times, how many sections would there be?
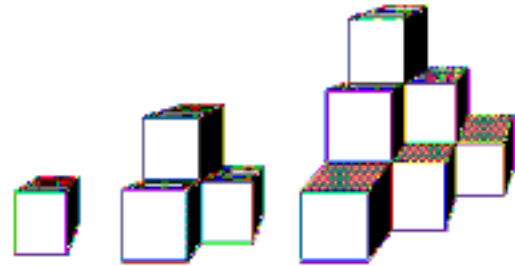
Answer: _____ sections.

How about folding it 8 times?

Answer: _____ sections.

★   3. If 24 out of 60 cows in the field are females, what proportion of the cows are female?

Answer: _____

★★★ 4. Look at the pattern of blocks below. If each cube has a volume of 5 cm^3, what is the volume of the block structure that would come next?

Answer: _____ cm^3

Strategy of the Month

What do you do if you have a problem that seems to be very complicated? It may have a lot of large numbers, too much information, or multiple conditions. One approach is to create a simpler problem like the one you need to solve. As you solve the easier problem, you may see the way to solve the more difficult one. Or you may discover a different process that will work with the harder problem. The trick is to be sure that your simpler problem is enough like the original one that the patterns or process you use will help you with the harder situation. ***Make a simpler problem*** *first as you solve this:*

The houses on Cox Avenue are numbered consecutively from 101 to 950. How many house numbers contain at least one digit 5?
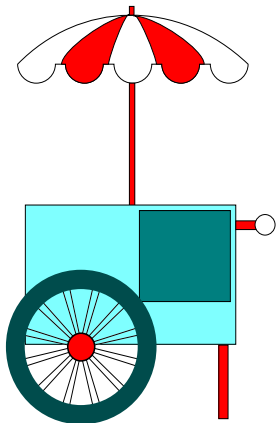
★★★ 7. The following average daily temperatures were recorded on 12 consecutive days:
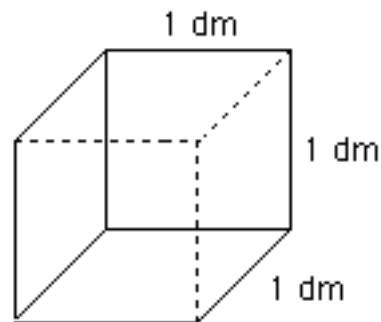
56 , 58 , 57 , 72 , 71 , 75 , 55 , 55 , 56 , 55 , 58 , 55

Construct a stem and leaf graph and then determine the mean and median temperature.

Mean _____ Median _____

What statement can you make about the trend of the temperature during these days?

★★★ 5. A bag contains only red, blue, and orange marbles. There is a total of 36 marbles in the bag. The probability of drawing a red marble is 2/3 and the probability of drawing a blue marble is 1/6. How many of each color are in the bag?

_____ orange _____ blue _____ red

★★ 8. How many centimeter cubes can be placed in the cube below.

★★ 6. Mary Martin owned one-half of a lemonade stand. She sold one-fourth of her share for $150. What was the value of the business?

Answer: _____

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Discussion of the problems...

1. **(2 chocolate, 1 vanilla, 1 strawberry; 1/2 carton of each left over)** There are five groups of five students in Mr. Higdon's class. In each group of five, three students prefer chocolate, one vanilla and one strawberry. That means 15 students in the class prefer chocolate, 5 prefer vanilla and 5 strawberry. Since a carton will serve 10 students he needs 2 of chocolate and one each of the other flavors. When the party is over there will be one-half carton of each left over. A diagram, or picture can help students "see" the situation.

2. **(64 sections, 256 sections)** Students can fold a paper up to five times before the folds become difficult if not impossible. Some may wish to try thinner or larger sheets but the thickness at five will always be an issue. A table of patterns can be started:

folds:	1	2	3	4	5	6	7	8
sections:	2	4	8	16	32	64	128	256

3. **(answers will vary)** The proportion 24:60 is one way to answer the question, but there are many equivalent forms: 12:30, 6:15, 2:5, 40:100 or 40%. Students can build families of equivalent proportions or ratios by using common multiples.

4. **(100 cm³)** Students may wish to use cubes to build these structures. The first in the series requires just one cube, the second adds a base of three cubes and elevates the first cube (4 total cubes and a volume 20 cm³. The third uses a base of six cubes and the structure before it is on top (10 total cubes with volume 50 cm³). The fourth figure would add a base of 10 cubes and the total structure would be 20 cubes for a volume of 100 cm³. The sequence 1, 4, 10, 20, . . . is increased by a consecutive triangular number at each stage.

5. **(6 orange, 6 blue, 24 red)** If the 36 marbles are divided into 6 groups, 1/6 would be six marbles so six are blue. When we divide the marbles into three groups, each will contain 12, so 24 or 2/3 are red. This accounts for 24 + 6 or 30 marbles, so the remaining six must be orange.

6. **($1200.00)** This is a good problem for the "draw a diagram" strategy.

Mary's Share
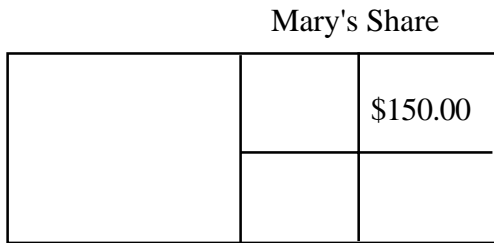
		$150.00

Viewing the one-fourth she sold for $150.00, students can compute the value of the entire business.

Lemonade Stand

7. **(mean 60.25, median 56.5 The temperature rose during the week and then dropped. It was warmer on the 3rd, 4th and 5th days than at any other time in the period, etc.)**

Stem and leaf plot: 5 | 5 5 5 5 6 6 7 8 8
                      6 |
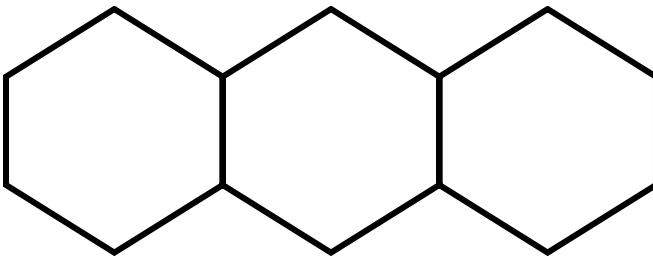   7 | 1 2 5

 Computing the median, some students may need assistance expressing the value between 56 and 57 as 56.5.

8. **(1000 cubes)** Students may wish to change 1 dm to 10 centimeters before beginning. The layers then become 100 square centimeters in area and it takes ten layers to fill the box.

Math*Stars*

★★★ 1. Decide where these polygons would go on the Venn diagram below and draw them in. Label the regions of the diagram to indicate which shapes belong to the region.

★★★★3. How many triangles in the figure below?
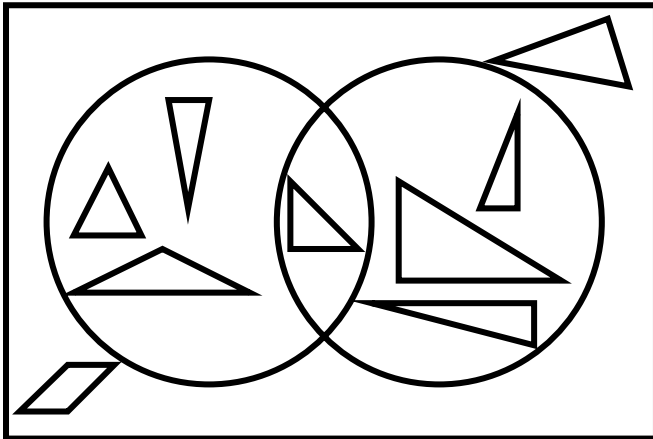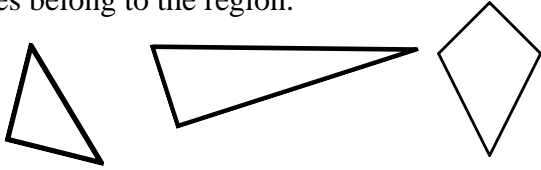
Answer: _____

★★   2. It takes six toothpicks to build one hexagon, 11 to build two hexagons, 16 to build three hexagons. How many toothpicks will it take to build ten hexagons?
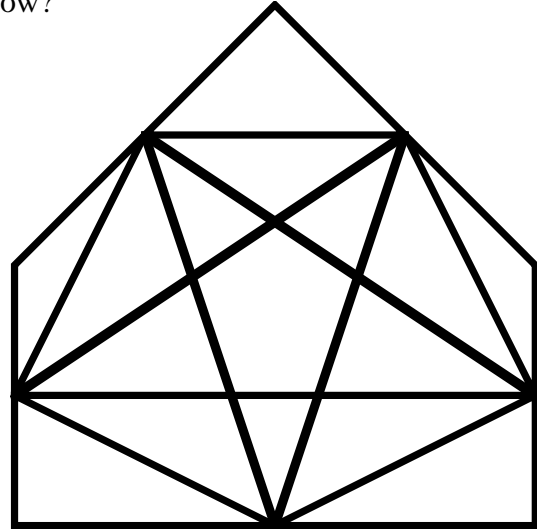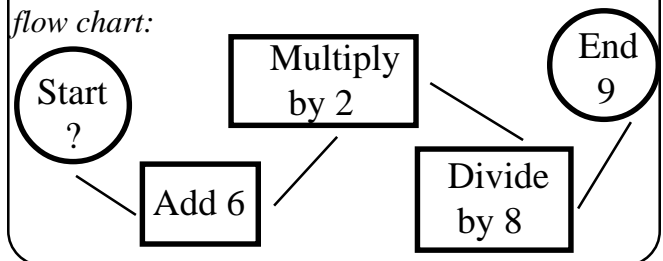
Strategy of the Month

*What if you know the result of a situation, but you don't know the beginning? For example, you might know that you end up with thirteen baseball cards after doing a certain number of trades and you want to figure out how many cards you had before the trading started. In that case you need to work backwards; you have to think about your actions in reverse order. This strategy works for any sequence of actions when you know the end result rather than the starting place. Try **working backwards** to find the starting number on this flow chart:*

Start ?

Multiply by 2

End 9

Add 6

Divide by 8

★★★ 7. The average weight of these bags of potatoes is 30 lbs. What is the weight of the unmarked bag?
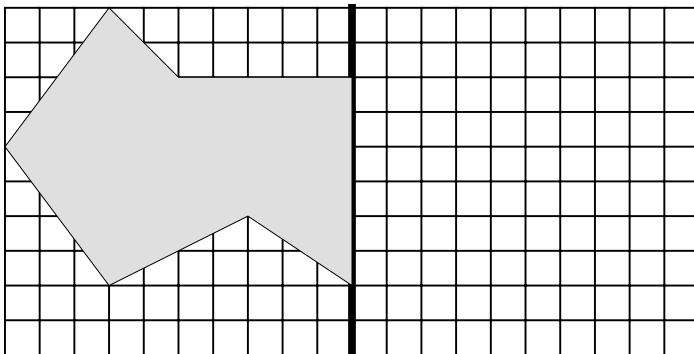

Answer: _____

★ 4. Find the sum of the counting numbers from 1 – 25. In other words, if $s = 1 + 2 + 3 \ldots + 24 + 25$; then find the value of s.
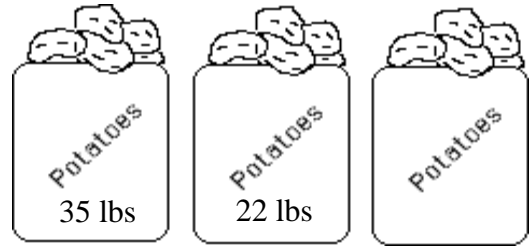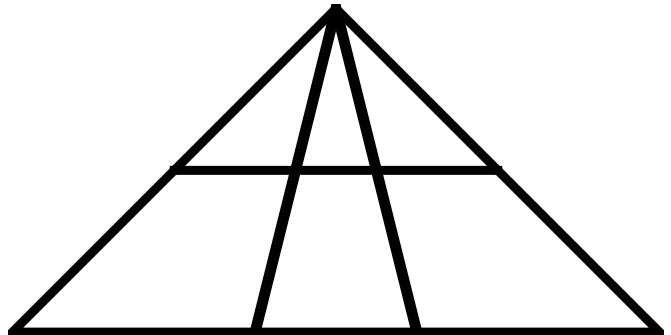
Answer: _____

★★★ 8. Find all of the triangles.

★★★ 5. Find the missing operations that make this number sentence true.

$$2.9 \,\boxed{}\, 1.42 + 6.03 \,\boxed{}\, 8.6 = 1.548$$

★★ 6. Draw the reflection of this design.

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Discussion of the problems...

1. **(The isosceles right triangle belongs in the intersection, the kite (quadrilateral) belongs outside the circles, the scalene right triangle belongs in the right hand circle.)**
 Labels:

polygons
isosceles triangles
right triangles
isosceles, right triangles
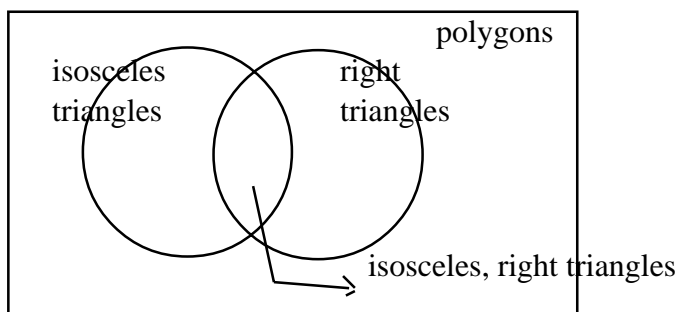
2. **(51 toothpicks)** Suggest that students make a table to see the relationship between the numbers in the series. One way to describe the pattern is: toothpicks = one more than five times the number of hexagons.

Hexagons:	1	2	3	4	5	6
Toothpicks:	6	11	16	21	26	31

3. **(40 triangles)** Perseverance and good organizational skills are important features of a successful problem solver. Making a table and counting by size or by region are just some of the ways students can approach this problem.

4. **(325)** Students will probably first approach this problem as a major addition exercise. Displaying the numbers and looking for patterns or ways to reduce the number of individual addends is one way to encourage some creative solutions. $1 + 2 + 3 + 4 + \ldots + 22 + 23 + 24 + 25$ All these sums are 26. How many are there? Are all the   addends accounted for? Would this technique work for other   problems like this?
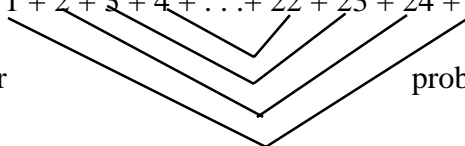
5. **(x , -)** Students need to look at the answer and note that it has three decimal places. This clue might help them determine that multiplication is part of the solution since it has three decimal places. After exploring the possible products, the other possibility involves reduction of the product. This leads to either subtraction or division. Using clues and determining possible operations and the rationale for their choice or elimination is an excellent situation for using number sense and determining students' understanding of the effect of the various operations.

6. Students may use a mirror or other device to see the reflection before attempting to draw it.
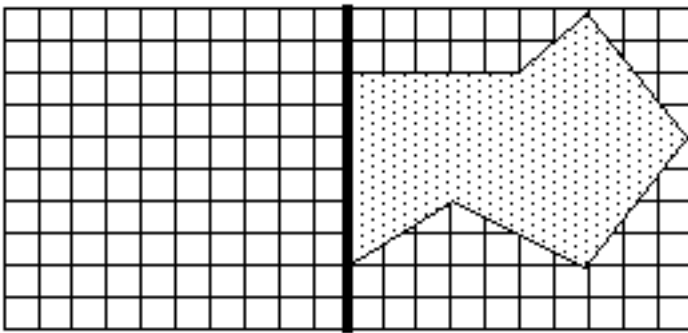
7. **(33)** Since the average makes all the bags equal in weight, the three bags weigh 3 x 30 or 90 pounds altogether. Now the problem becomes $35 + 22 + N = 90$.

8. **(12)** A good strategy for determining the number of triangles involves organizing the count. Starting with the smallest and proceeding to the larger sizes, using colored pencils, or drawing and shading the triangles will help students keep track of their discoveries.

Math*Stars*

★★★★1. Cut the pizza using three straight cuts so that each pepperoni is on a slice by itself. (Slices do not have to be congruent.)
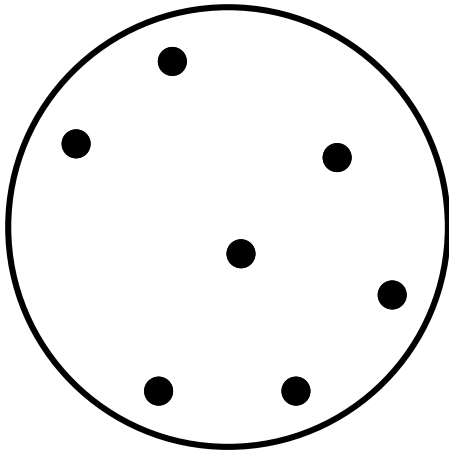
★★   2. All the fifth grade classes are going on a field trip. They will take three buses that hold 54 passengers each. If the buses are full and there are seven chaperones going, how many students are going?

Answer: _____

★   3. What is the value of ⌘ if

(⌘ x 8) + 2 = 26

Answer: _____

★★★ 4. The perimeter of a rectangle is 120 meters. The length is twice the width. Find the area of the rectangle.

Answer: _____

Strategy of the Month

*You have tried many ways to solve problems this year. Already you know that when one strategy does not lead you to a solution, you back up and try something else. Sometimes you can find a smaller problem inside the larger one that must be solved first. Sometimes you need to think about the information that is missing rather than what is there. Sometimes you need to read the problem again and look for a different point of view. Sometimes you need to tell your brain to try to think about the problem in an entirely different way - perhaps a way you have never used before. Looking for different ways to solve problems is like brainstorming. Try to solve this problem. You may need to **change your point of view** by asking, "Do all of the lines have to stay within the square formed by the dots?"*

Draw 4 line segments through all 9 dots without lifting your pencil or pen.
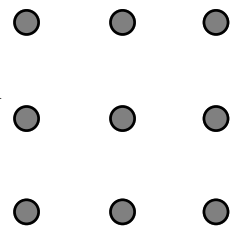
★★★ 5. In a book with 382 pages, how many 3's would be used to number the pages?
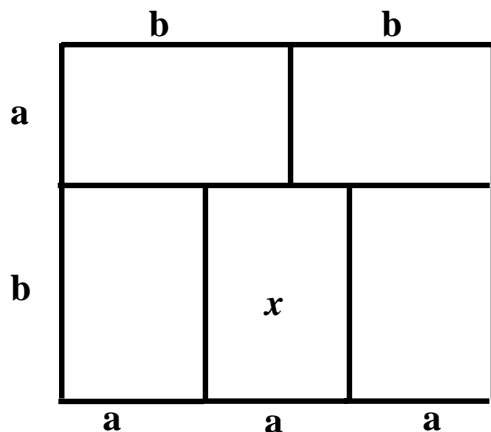
Answer: _____

★★ 6. How many numbers can be made from the following digits if each digit is used one time?
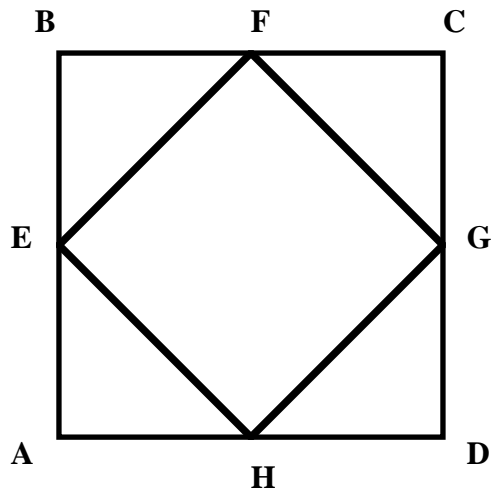
$$3 \quad 5 \quad 9 \quad 6$$

Answer: _____

★★★ 7. The large rectangle is divided into five congruent rectangles. If $a + b = 10$, find the area of the rectangular region labeled x.

★★★ 8. Figure ABCD is a square with sides that measure four. E, F, G, and H are the midpoints of the sides. What is the area of ◻EFGH?

Answer: _____

★★ 9. If each letter represents a different digit, what is the value of each letter?

$$\begin{array}{r} A\,B\,C \\ \times \quad\ C \\ \hline D\,B\,C \end{array}$$
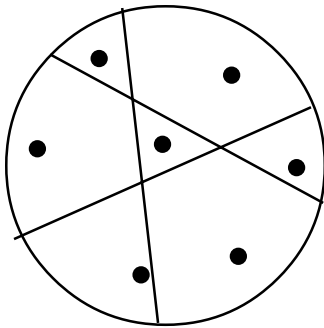
About these newsletters...

 The purpose of the MathStars Newsletters is to challenge students beyond the classroom setting. Good problems can inspire curiosity about number relationships and geometric properties. It is hoped that in accepting the challenge of mathematical problem solving, students, their parents, and their teachers will be led to explore new mathematical horizons.

As with all good problems, the solutions and strategies suggested are merely a sample of what you and your students may discover. Enjoy!!

Discussion of the problems...

1.

2. **(155)** The first step is to find out how many people are on the three buses, i.e., 3 x 54 or 162 riders. If seven are chaperones then 155 are students.

3. **(3)** A good example of working backwards. If the last operation is adding two, its opposite would be subtracting 2. Before that you multiplied by eight, and the opposite or inverse operation would be division by eight. After students have explored and discussed this problem, they may wish to make up equations for their classmates to solve.

4. **(800 square meters)** Guess and check is a good strategy here. Making a chart or list to organize the guesses will help focus students' work. The length is twice the width and the perimeter is 120. Some students may note that length plus width totals 60.

5. **(161)** Students can again make a table or chart to organize their counting.

Nos.	1-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	91-100	101-200	201-300	301-382
3's	1	1	2	10	1	1	1	1	1	1	20	21	82 + 18

6. **(24)** Organizing their choices will help students keep track of the various possibilities. Tree diagrams are helpful and can give students a new strategy.

This scheme gives six numbers with 3 in thousands place. Next would be six with 5 as the lead digit, etc.
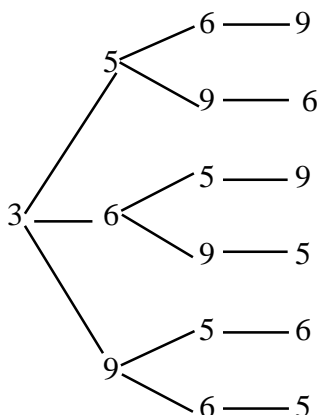
Another strategy could be to list the numbers by size, starting with the smallest, 3,569 and proceeding to the largest, 9,653. A logical approach could ask the question, "How many choices are there for thousands place? How many for hundreds, for tens, for ones?", i.e., 4 x 3 x 2 x 1 = 24.

7. **(24 square units)** Since $a + b = 10$, students need to examine the various addends with one other piece of information: $3a = 2b$. Guess and check will give the values of a and b respectively, as 4 and 6.

8. **(8 square units)** The smaller square has area one-half the larger square. Students can demonstrate this by cutting the triangular pieces and fitting them into the square, or by computing the area of the four triangles and subtracting it from the larger area.

9. **(A = 1, B = 2, C = 5, D = 6, or A = 1, B = 7, C = 5, D = 8)** Students can begin by reasoning that C is one, five or six. Trial and error eliminates one and six, and students can proceed to the other digits from there.